Harvard Undergraduate Science Olympiad 2025

Final Round

Physics Syllabus: 7th-8th Grade
Answer Key

Part A Multiple Choice

1. A
2. B
3. C
4. B
5. D
6. A
7. B
8. A

9. C
10. B
11. C
12. B
13. C
14. B
15. C
16. C
17. B
18. A
19. B
20. B

Part B Free Response Questions

1. Apply Snell's law at the first surface (from air to glass):

$$n_{\text{air}} \sin\theta_{in} = n_{\text{glass}} \sin\theta_r \quad \Rightarrow \quad 1.00 \sin 30^\circ = 1.50 \sin\theta_r$$

$$\sin\theta_r = \frac{1}{3} \;\Rightarrow\; \theta_r \approx 19.47^\circ.$$

   (a) At the second surface (frm glass to air),

$$1.50 \sin\theta_r = 1.00 \sin\theta_{\text{out}} \;\Rightarrow\; \sin\theta_{\text{out}} = \sin 30^\circ \;\Rightarrow\; \theta_{\text{out}} = 30^\circ,$$

   so the emerging ray is parallel to the incoming ray.

   (b) The lateral shift for a parallel slab is

$$\Delta = t\,\frac{\sin(\theta_{in} - \theta_r)}{\cos\theta_r}.$$

   With $t = 2.0\,\text{cm}$, $\theta_{in} = 30^\circ$, and $\theta_r \approx 19.47^\circ$,

$$\Delta \approx 2.0\,\frac{\sin(10.53^\circ)}{\cos(19.47^\circ)} \approx 2.0\,\frac{0.183}{0.943} \approx 0.388\,\text{cm} \approx 3.9\,\text{mm}.$$

2. The bottle holds 450 mL total. With 1 : 2 parts by volume:

$$V_1 = 150\,\mathrm{mL}, \qquad V_2 = 300\,\mathrm{mL}.$$

(a) Room-temperature water: $m_c = \rho V_1 = 1000\mathrm{kg/m^3}(150 \times 10^{-6}\mathrm{m^3}) = 0.150\,\mathrm{kg}$ at 20°C. Boiling water: $m_h = \rho V_2 = 1000\mathrm{kg/m^3}(300 \times 10^{-6}\mathrm{m^3}) = 0.300\,\mathrm{kg}$ at 100°C. Because of conservation of energy, we have that the heat assimilated by the cold water and water bottle is equal to the heat released by the warm water:

$$m_h c(100 - T_f) = m_c c(T_f - 20) + C_{\mathrm{bottle}}(T_f - 20)$$

$$T_f = \frac{m_h c \cdot 100 + (m_c c + C_{\mathrm{bottle}}) \cdot 20}{(m_h + m_c)c + C_{\mathrm{bottle}}}$$

With $c = 4200\,\mathrm{J/(kg \cdot K)}$ and $C_{\mathrm{bottle}} = 20\,\mathrm{J/K}$,

$$T_f \approx 72.8°\mathrm{C}.$$

(b) Water: $m_w = \rho(300\,\mathrm{mL}) = 0.300\,\mathrm{kg}$ at 20°C. Ice volume is 150 mL, so

$$m_i = \rho_{\mathrm{ice}} V_1 = 900\mathrm{kg/m^3}(150 \times 10^{-6}\mathrm{m^3}) = 0.135\,\mathrm{kg} \quad \text{at } 0°\mathrm{C}.$$

First check if all the ice can melt. The maximum heat available by cooling the water and bottle from 20°C to 0°C is

$$Q_{\mathrm{max}} = (m_w c + C_{\mathrm{bottle}})\,20 = [0.300\mathrm{kg} \cdot 4200\mathrm{J/(K \cdot kg)} + 20\mathrm{J/(K \cdot kg)}] \cdot 20\mathrm{K} \approx 2.56 \times 10^4\,\mathrm{J}.$$

Heat needed to melt all the ice:

$$Q_{\mathrm{melt}} = m_i L = 0.135\mathrm{kg} \cdot 3.34 \times 10^5\mathrm{J/kg} \approx 4.51 \times 10^4\,\mathrm{J}.$$

Since $Q_{\mathrm{max}} < Q_{\mathrm{melt}}$, not all the ice melts and therefore

$$T_f = 0°\mathrm{C}.$$

Melted ice mass:

$$m_{\mathrm{melt}} = \frac{Q_{\mathrm{max}}}{L} \approx \frac{2.56 \times 10^4\mathrm{J}}{3.34 \times 10^5\mathrm{J}} \approx 7.66 \times 10^{-2}\,\mathrm{kg} = 76.6\,\mathrm{g}.$$

Ice left:

$$m_{\mathrm{left}} = m_i - m_{\mathrm{melt}} \approx 0.135\mathrm{kg} - 0.0766\mathrm{kg} \approx 0.0584\,\mathrm{kg} = 58.4\,\mathrm{g}.$$

3. (a) At equilibrium,

$$mg = kx_0 \quad \Rightarrow \quad x_0 = \frac{mg}{k}.$$

$$x_0 = \frac{(0.50\,\mathrm{kg})(10\,\mathrm{m/s^2})}{40\,\mathrm{N/m}} = 0.125\,\mathrm{m}.$$

(b) Using energy about the equilibrium point,

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = A\sqrt{\frac{k}{m}}.$$

$$v = (0.20\,\mathrm{m})\sqrt{\frac{40\,\mathrm{N/m}}{0.50\,\mathrm{kg}}} = (0.20\,\mathrm{m})\sqrt{80\,\mathrm{s^{-2}}} \approx 1.79\,\mathrm{m/s}.$$

(c) Again using energy about equilibrium,

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k}{m}\,(A^2 - x^2)}.$$

For $x = 0.10\,\mathrm{m}$,

$$v = \sqrt{\frac{40\,\mathrm{N/m}}{0.50\,\mathrm{kg}}\,[(0.20\,\mathrm{m})^2 - (0.10\,\mathrm{m})^2]} = \sqrt{80\,\mathrm{s^{-2}} \cdot 0.03\,\mathrm{m^2}} \approx 1.55\,\mathrm{m/s}.$$